

## Bridgewater Primary School

### Calculation Guidelines

(updated Spring 2015)

#### **Introduction**

Children are introduced to the processes of calculation through practical, oral and mental activities. As children begin to understand the underlying ideas, they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved. Over time children learn how to use models and images, such as empty number lines, to support their mental and informal written methods of calculation. As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally. By the end of Year 6 children are equipped with mental, written and calculator methods that they understand and can use correctly.

The overall aim is that when children leave Bridgewater they:

- have a secure knowledge of number facts and a good understanding of the four operations;
- are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;
- make use of diagrams and informal notes to help record steps and part-answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;
- use a calculator effectively, using their mental skills to monitor the process, check the steps involved and decide if the numbers displayed make sense.

**When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy. At whatever stage in their learning, and whatever method is being used, it must still be underpinned by a secure and appropriate knowledge of number facts, along with those mental skills that are needed to carry out the process and judge if it was successful.**

THE FOLLOWING ARE STANDARDS THAT WE EXPECT THE MAJORITY OF CHILDREN TO ACHIEVE

## Addition

### Y1

#### Rapid Recall of

- all pairs of number bonds within 20;
- all pairs of numbers with a total of 10, eg  $3 + 7$ ;
- addition doubles of all numbers to at least 10, eg  $4 + 4$ .

#### Mental strategies

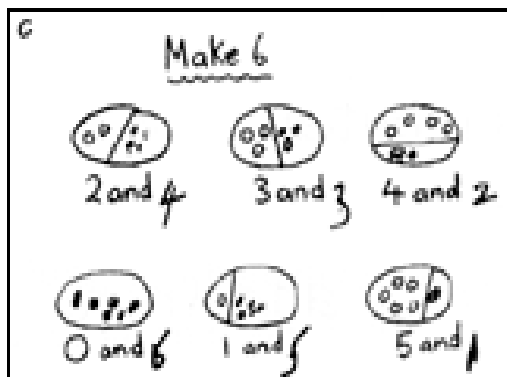
- count on in ones;
- reorder numbers in a calculation;
- begin to bridge through 10, and later 20, when adding a single-digit number;
- use known number facts and place value to add pairs of single-digit numbers;
- add 9 to single-digit numbers by adding 10 then subtracting 1;
- identify near doubles, using doubles already known;
- use patterns of similar calculations.

#### Mental Calculations

- add a single-digit to a single-digit, without crossing 10, eg  $4 + 5$ ;
- add a single-digit to 10;
- add a single-digit to a 'teens' number, without crossing 20 or 10, eg  $13 + 5$ ;
- doubles of all numbers to 10, eg  $8 + 8$ , double 6.

#### Written Step 1 (YR and Y1)

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.

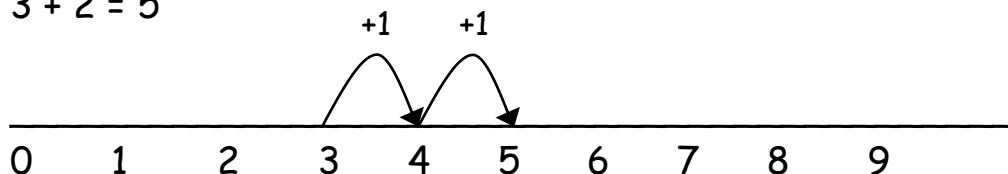


Remember! Children need to move on to being able to count on from one of the numbers.

They use number lines and tracks as well as other practical resources to support calculation and teachers *demonstrate* the use of the number line.

*Teachers demonstrate:*

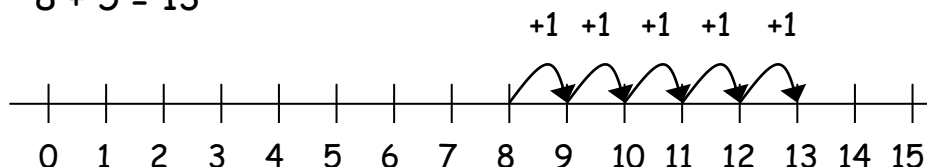
$$3 + 2 = 5$$



Children then begin to use numbered lines to support their own calculations using a numbered line to count on in jumps of one.

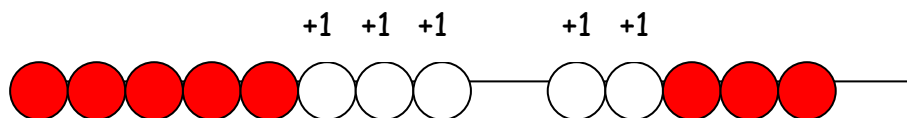
e.g. Child's recording

$$8 + 5 = 13$$



Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3 (Use ITP - Interactive Teaching Programs).

$$8 + 5 = 13$$



## Year 2

### Rapid Recall of

- addition facts for all numbers to at least 20 fluently;
- all pairs of numbers with a total of 20, eg  $13 + 7$ ;
- all pairs of multiples of 10 with a total of 100, eg  $30 + 70$ ;

### Mental strategies

- count on in tens or ones;
- reorder numbers in a calculation;
- add three small numbers by putting the largest number first and/or find a pair totaling 10;
- partition additions into tens and units then recombine;
- bridge through 10 or 20;
- use known number facts and place value to add pairs of numbers;
- partition into '5 and a bit' when adding 6, 7, 8 or 9, then recombine;
- add 9, 19, 11 or 21 by rounding and compensating;
- identify near doubles;

- use patterns of similar calculations;
- use the relationship between addition and subtraction;
- derive and use related facts to 100

### Mental Calculations

- add a single-digit to a single-digit, without crossing 10, e.g.  $4 + 5$ ;
- add a single-digit to 10;
- add a single-digit to a 'teens' number, without crossing 20 or 10, eg  $13 + 5$ ;
- doubles of all numbers to 10, eg  $8 + 8$ , double 6, then teen numbers.

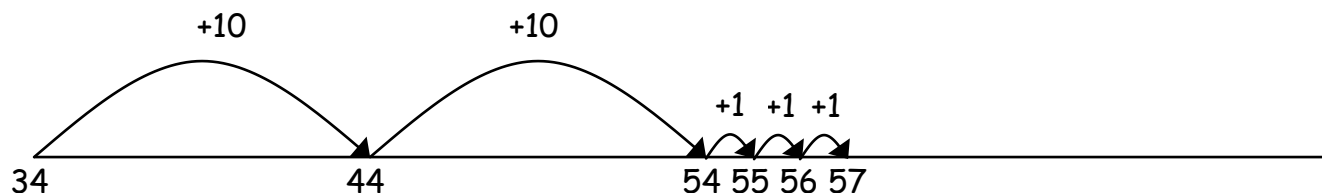
### Written (Y2)

Children will continue to use concrete objects including structured apparatus such as Dienes and /or cubes in blocks of ten and pictorial representations.

Children will progress to use 'empty number lines' themselves starting with the larger number and counting on.

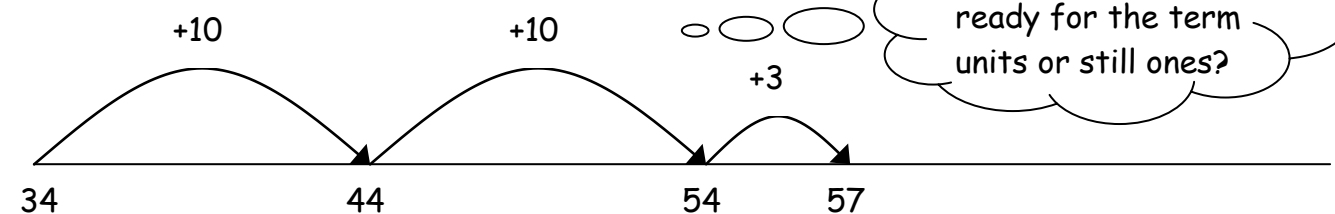
- ✓ First counting on in tens and ones (units).

$$34 + 23 = 57$$



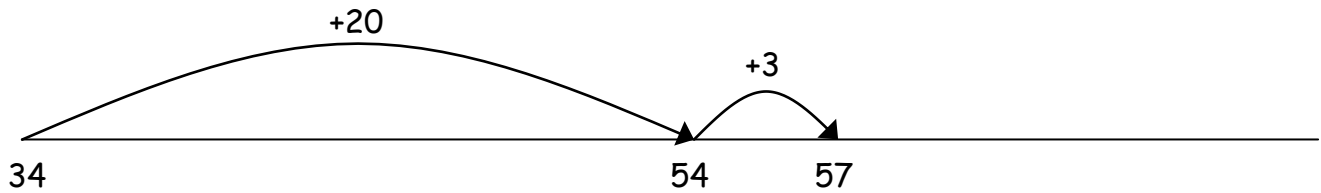
- ✓ Then helping children to become more efficient by adding the units in one jump (by using the known fact  $4 + 3 = 7$ ).

$$34 + 23 = 57$$



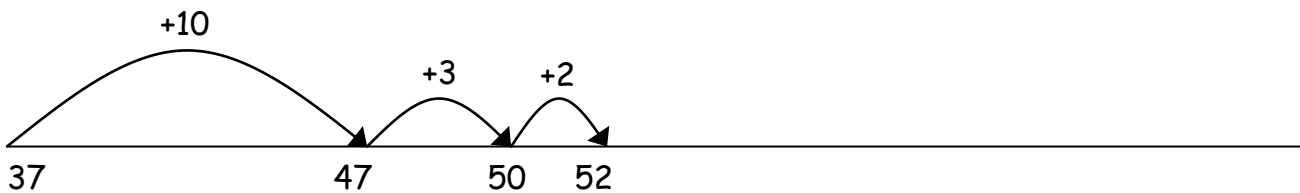
✓ Followed by adding the tens in one jump and the units in one jump.

$$34 + 23 = 57$$



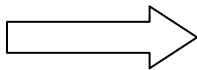
✓ Bridging through ten can help children become more efficient.

$$37 + 15 = 52$$



Some children may progress to columnar recording of addition which would be initially accompanied through base 10 apparatus.

$$\begin{array}{r} 42 \\ + 13 \\ \hline \end{array}$$



$$\begin{array}{r} 40 + 2 \\ + 10 + 3 \\ \hline 50 + 5 = 55 \end{array}$$

Children need to see the value of each digit.

### Year 3

#### Rapid Recall of

- addition facts for all numbers to 100;
- all pairs of multiples of 100 with a total of 1000;
- all pairs of multiples of 5 with a total of 100;

#### Mental strategies

- count on in tens or ones;
- reorder numbers in a calculation;
- add three or four small numbers by putting the largest number first and/or by finding pairs totalling 9, 10 or 11;
- partition into tens and units then recombine;
- bridge through a multiple of 10, then adjust;
- use knowledge of number facts and place value to add pairs of numbers;

- partition into '5 and a bit' when adding 6, 7, 8 or 9;
- add mentally a 'near multiple of 10' to a two-digit number;
- identify near doubles;
- use patterns of similar calculations.

### Mental Calculations

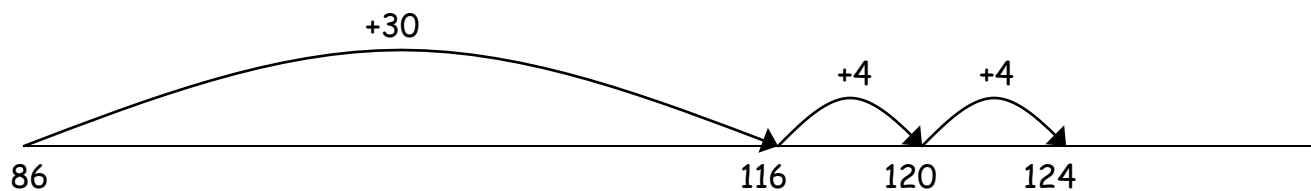
- find what must be added to any multiple of 100 to make 1000, eg  $300 + ? = 1000$ ;
- add any pair of two-digit numbers, without crossing a tens boundary or 100, eg  $33 + 45$ ;
- add any single-digit to any two-digit number, including crossing the tens boundary, eg  $67 + 5$ ;
- find what must be added to any two-digit number to make the next higher/lower multiple of 10, eg  $64 + ? = 70$ ;
- find what must be added to any three-digit number to make the next higher/lower multiple of 10, eg  $647 + ? = 650$ ;
- add 3-digit numbers and ones (units);
- add 3-digit numbers and tens;
- add 3-digit numbers and hundreds.

### Step 3 (Y3) Written

Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate and, as they are ready, move towards columnar recording.

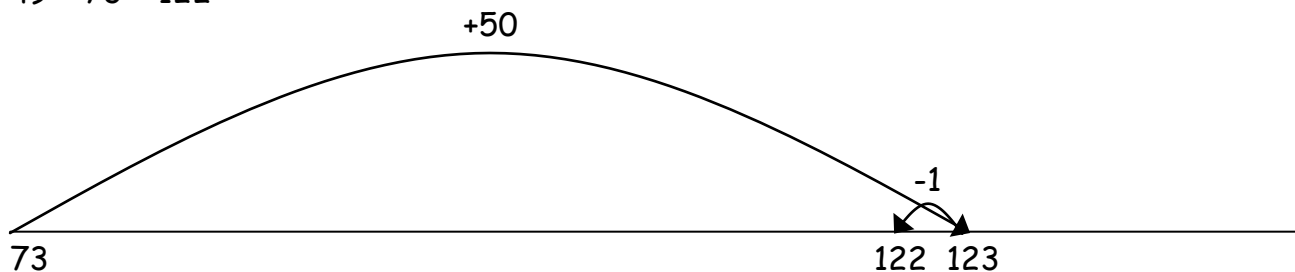
- ✓ Count on from the largest number irrespective of the order of the calculation.

$$38 + 86 = 124$$



- ✓ Rounding and adjusting

$$49 + 73 = 122$$



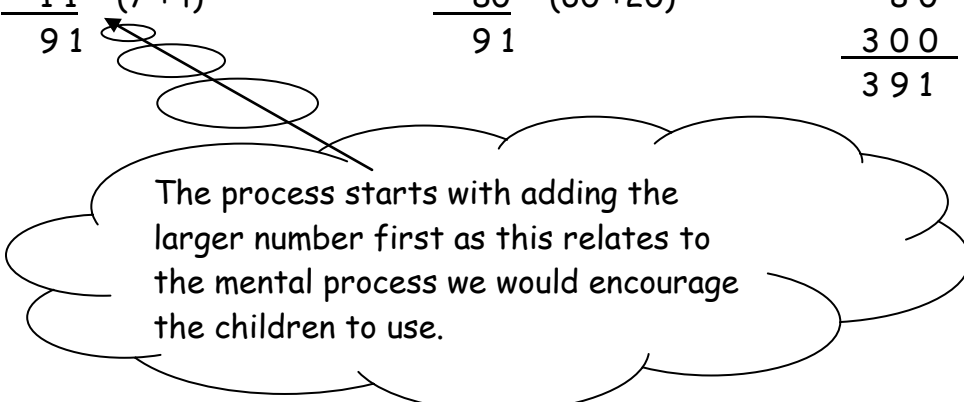
Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

At Bridgewater we will model adding the most significant digits first and then quickly move to adding the least significant digits first in preparation for 'carrying'.

$$\begin{array}{r} 67 \\ + 24 \\ \hline 80 \quad (60 + 20) \\ 11 \quad (7 + 4) \\ \hline 91 \end{array}$$

$$\begin{array}{r} 67 \\ + 24 \\ \hline 11 \quad (7 + 4) \\ 80 \quad (60 + 20) \\ \hline 91 \end{array}$$

$$\begin{array}{r} 267 \\ + 124 \\ \hline 11 \quad (7 + 4) \\ 80 \quad (60 + 20) \\ 300 \quad (200 + 100) \\ \hline 391 \end{array}$$



The process starts with adding the larger number first as this relates to the mental process we would encourage the children to use.

## Year 4

### Rapid Recall of

#### Mental strategies

- count on in repeated steps of 1, 10 and 100;
- count up through the next multiple of 10, 100 or 1000;
- reorder numbers in a calculation;
- add 3 or 4 small numbers, finding pairs totalling 10;
- add three two-digit multiples of 10;
- partition into tens and units, adding the tens first;
- bridge through 100;
- use knowledge of number facts and place value to add any pair of two-digit numbers;
- add 9, 19, 29, 11, 21 or 31 by rounding and compensating;
- add the nearest multiple of 10 then adjust;
- continue to use the relationship between addition and subtraction.

#### Mental Calculations

- find what must be added to any two-digit number to make 100, eg  $37 + ? = 100$ ;
- add any pair of two-digit numbers, eg  $38 + 85$ ;
- find out what must be added to any two- or three-digit number to make the next higher/lower multiple of 100, eg  $374 + ? = 400$ .

## Y4 Written

From this, children will begin to carry below the line with up to four digits.

$$\begin{array}{r} 625 \\ + 48 \\ \hline 673 \\ 1 \end{array}$$

$$\begin{array}{r} 783 \\ + 42 \\ \hline 825 \\ 1 \end{array}$$

$$\begin{array}{r} 367 \\ + 85 \\ \hline 452 \\ 11 \end{array} \quad \begin{array}{r} 6534 \\ + 2376 \\ \hline 8910 \\ 11 \end{array}$$

*Using similar methods, children will:*

- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds;*
- ✓ *know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + 78p.*

## Year 5

### Rapid Recall of

#### Mental strategies

- count up through the next multiple of 10, 100 or 1000;
- reorder numbers in a calculation;
- partition into hundreds, tens and units, adding the most significant digit first;
- use known number facts and place value to add pairs of three-digit multiples of 10 and two-digit numbers with one decimal place;
- add the nearest multiple of 10 or 100 then adjust;
- identify near doubles;
- add several numbers;
- develop further the relationship between addition and subtraction.

#### Mental Calculations

- add any pair of three-digit multiples of 10, eg  $570 + 250$ ;
- find what must be added to a decimal fraction with units and tenths to make the next higher whole number, eg  $4.3 + ? = 5$ ;
- add any pair of decimal fractions each with units and tenths, or each with tenths and hundredths, eg  $5.7 + 2.5$ ;
- add mentally numbers with more than four digits.



## Step 5 Written (Y5)

Children should extend the carrying method to numbers with more than four digits.

$$\begin{array}{r} 587 \\ + 475 \\ \hline 1062 \\ \hline 1 \quad 1 \end{array}$$

$$\begin{array}{r} 3587 \\ + 675 \\ \hline 4262 \\ \hline 1 \quad 1 \quad 1 \end{array}$$

*Using similar methods, children will:*

- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more decimal fractions with up to three digits and the same number of decimal places;*
- ✓ *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m - 280 cm.*

## Year 6

### Rapid Recall of

#### Mental strategies

- consolidate all strategies from previous years;
- use knowledge of number facts and place value to add three-digit multiples of 10 and two-digit numbers with one decimal place;
- add the nearest multiple of 10, 100 or 1000, then adjust;
- continue to use the relationship between addition and subtraction.

#### Mental Calculations

## Step 6 Written (Y6)

Children should extend the carrying method to a number with any number of digits working with larger numbers.

$$\begin{array}{r} 7648 \\ + 1486 \\ \hline 9134 \\ \hline 1 \quad 1 \quad 1 \end{array}$$

$$\begin{array}{r} 6584 \\ + 5848 \\ \hline 12432 \\ \hline 1 \quad 1 \quad 1 \quad 1 \end{array}$$

$$\begin{array}{r} 42 \\ 6432 \\ 786 \\ 3 \\ + 4681 \\ \hline 11944 \\ \hline 1 \quad 1 \quad 2 \quad 1 \end{array}$$

*Using similar methods, children will:*

- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more decimal fractions with up to four digits and either one or two decimal places;*
- ✓ *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g.  $401.2 + 26.85 + 0.71$ .*

+ - + - + - + - + - + - +

## Subtraction

### Y1

#### Rapid Recall of

-subtraction facts for all numbers to at least 10 then 20.

#### Mental strategies

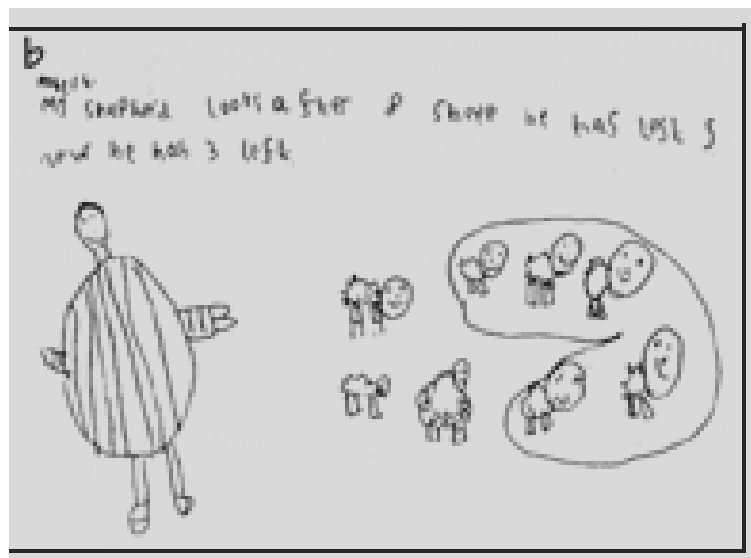
- count back in ones;
- reorder numbers in a calculation;
- use known number facts and place value to subtract pairs of single-digit numbers;
- use patterns of similar calculations.

#### Mental Calculations

- subtract a single digit from a single digit, without crossing 10, eg  $8 - 3$ ;
- subtract a single digit from 10;
- subtract a single digit from a 'teens' number, without crossing 20 or 10, eg  $17 - 3$ .

#### Written YR and Y1

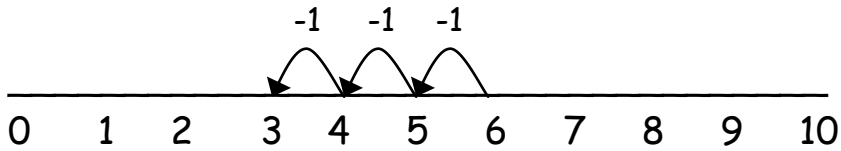
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation and they are supported in this by the teacher using a variety of models and images. They develop ways of recording calculations using pictures etc.



They use number lines and practical resources to support calculation. Teachers *demonstrate* the use of the number line.

Teachers demonstrate:

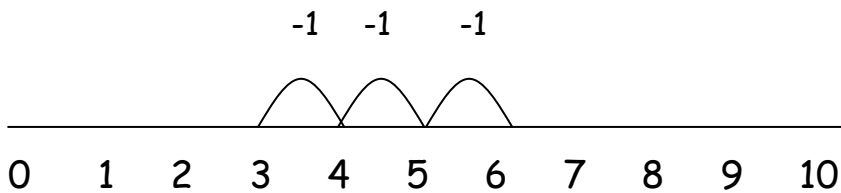
$$6 - 3 = 3$$



The number line should also be used to show that  $6 - 3$  means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart.

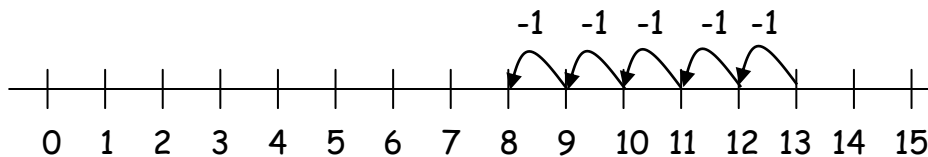
e.g. Child's recording

$$6 - 3 = 3$$



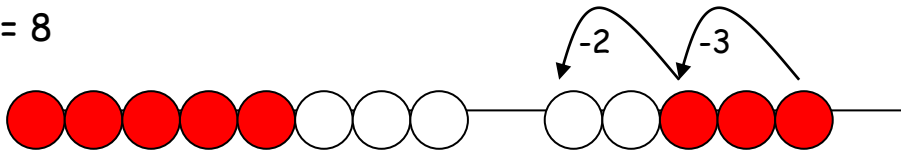
Children then begin to use numbered lines to support their own calculations—using a numbered line to count back in ones (units).

$$13 - 5 = 8$$



Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.

$$13 - 5 = 8$$



## Year 2

### Rapid Recall of

- subtraction facts for all numbers to at least 20 and derive related facts up to 100.

### Mental strategies

- count back in tens or ones;
- find a small difference by counting up from the smaller to the larger number;
- reorder numbers in a calculation;

- bridge through 10 or 20;
- use known number facts and place value to subtract pairs of numbers;
- subtract 9, 19, 11 or 21 by rounding and compensating;
- use patterns of similar calculations;
- use the relationship between addition and subtraction.

### Mental Calculations

- subtract a single digit from a single digit, without crossing 10, eg  $8 - 3$ ;
- subtract a single digit from 10;
- subtract a single digit from a 'teens' number, without crossing 20 or 10, eg  $17 - 3$ ;
- subtract a single-digit number from a two-digit number;
- subtract a multiple of 10 from a two-digit number;
- subtract a two-digit number from a two-digit number.

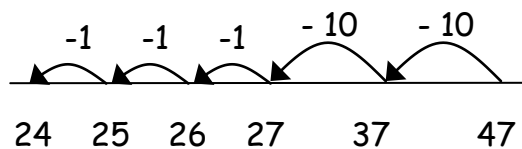
### Written Y2

Children will begin to use empty number lines to support calculations. Children can be encouraged to draw their own as appropriate.

#### Counting back

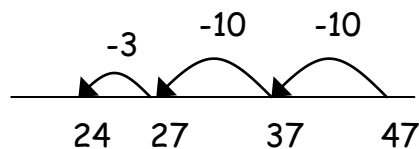
- ✓ First counting back in tens and ones (units).

$$47 - 23 = 24$$



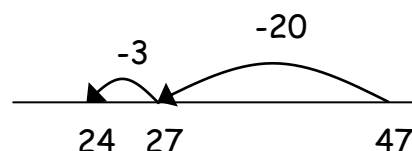
- ✓ Then helping children to become more efficient by subtracting the units in one jump (by using the known fact  $7 - 3 = 4$ ).

$$47 - 23 = 24$$



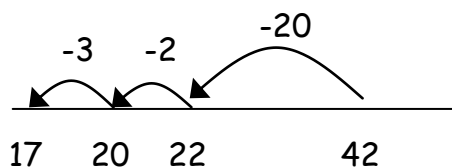
- ✓ Subtracting the tens in one jump and the units in one jump.

$$47 - 23 = 24$$



- ✓ Bridging through ten can help children become more efficient.

$$42 - 25 = 17$$

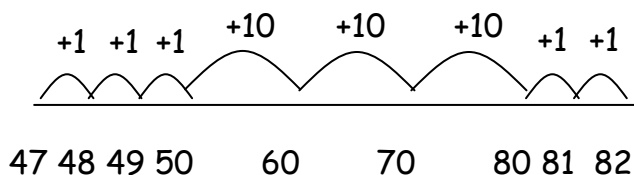


### Counting on

If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on.

Count up from 47 to 82 in jumps of 10 and jumps of 1.

$$82 - 47$$



Help children to become more efficient with counting on by:

- ✓ adding the units in one jump;
- ✓ adding the tens in one jump and the units in one jump;
- ✓ bridging through ten.

Some children may progress to columnar recording of subtraction, which would be initially accompanied through base 10 apparatus.

$$\begin{array}{r} 23 \\ - 14 \\ \hline \end{array}$$

$$\begin{array}{r} 20 + 3 \\ - 10 + 4 \\ \hline 30 + 7 = 37 \end{array}$$

Children need to see the value of each digit.

## Year 3

### Rapid Recall of

- subtraction facts for all numbers to 100.

### Mental strategies

- count back in tens or ones;
- find a small difference by counting up from the smaller to the larger number;
- partition into tens and units then recombine;
- bridge through a multiple of 10, then adjust;
- use knowledge of number facts and place value to subtract pairs of numbers;
- subtract mentally a 'near multiple of 10' from a two-digit number;
- use patterns of similar calculations;
- say or write a subtraction statement corresponding to a given addition statement.

### Mental Calculations

- subtract any pair of two-digit numbers, without crossing a tens boundary or 100, eg  $87 - 2$ ;
- subtract any single-digit from any two-digit number, including crossing the tens boundary, eg  $82 - 7$ ;
- find what must be subtracted from any two-digit number to make the next higher/lower multiple of 10, eg  $56 - ? = 50$ ;
- subtract any three-digit number from any three-digit number when the difference is less than 10, eg  $458 - 451$ , or  $603 - 597$ ;
- find what must be subtracted from any three-digit number to make the next higher/lower multiple of 10, eg  $246 - ? = 240$ ;
- subtract a single-digit number from a three-digit number;
- subtract a multiple of 10 from a three-digit number;
- subtract a multiple of 100 from a three-digit number.

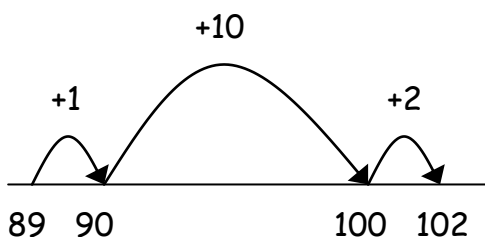
### Step 3 Written (Y3)

Children will continue to use empty number lines with increasingly large numbers.

Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, **counting on** using a number line could be used.

$$102 - 89 = 13$$



### Partitioning and decomposition

This process could be demonstrated using arrow cards to show the partitioning and base 10 materials to show the decomposition of the number.

$$\begin{array}{r} 80 + 9 \\ - 50 + 7 \\ \hline 30 + 2 \end{array}$$

$$\begin{array}{r} 89 \\ - 57 \\ \hline 32 \end{array}$$

*Initially, the children will be taught using examples that do not need the children to exchange.*

**From this the children will begin to exchange.**

$$74 - 27 =$$

$$\begin{array}{r} 70 \quad 4 \\ - 20 \quad 7 \\ \hline \end{array}$$

This leads to...

$$\begin{array}{r} 60 \quad 14 \\ \cancel{70} \quad 4 \\ - 20 \quad 7 \\ \hline 40 + 7 \end{array}$$

$$\begin{array}{r} 6 \quad 14 \\ \cancel{7} \quad 4 \\ - 2 \quad 7 \\ \hline 4 \quad 7 \end{array}$$

The calculation should be read as, e.g., 70 subtract

The calculation should be read as, e.g., take 7 from 4.

*Children should know that units line up under units, tens under tens, and so on.*

Children should become confident with working with numbers with 3 digits.



## Year 4

### Rapid Recall of

#### Mental strategies

- count back in repeated steps of 1, 10 and 100;
- reorder numbers in a calculation;
- use knowledge of number facts and place value to subtract any pair of two-digit numbers;
- subtract 9, 19, 29, 11, 21 or 31 by rounding and compensating;
- subtract the nearest multiple of 10 then adjust;
- identify near doubles;
- continue to use the relationship between addition and subtraction.

#### Mental Calculations

- subtract any pair of two-digit numbers, eg  $92 - 47$ ;
- find out what must be subtracted from any two- or three-digit number to make the next higher/lower multiple of 100, eg  $826 - ? = 800$ ;
- subtract any four-digit number from any four-digit number when the difference is small, eg  $3641 - 3628$ ,  $6002 - 5991$ ;

### Written Y4

#### Decomposition

$$\begin{array}{r} 614.14 \\ \cancel{7}54 \\ - \quad 86 \\ \hline 668 \end{array}$$

*Children should:*

- ✓ *subtract numbers with up to 4 digits using a formal written method;*
- ✓ *be able to subtract numbers with different numbers of digits;*
- ✓ *using this method, children should also begin to find the difference between two three-digit sums of money, with or without 'adjustment' from the pence to the pounds;*
- ✓ *know that decimal points should line up under each other.*

*For example:*

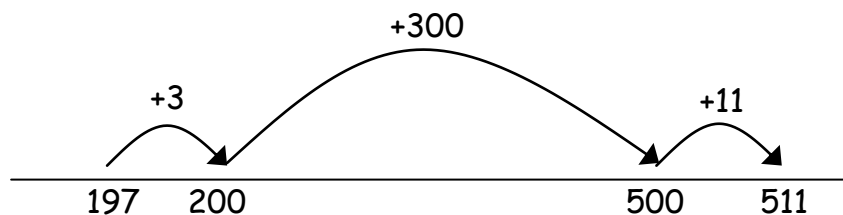
$$\begin{array}{r} 8.15 \\ \pounds 8.15 \\ - \pounds 4.38 \\ \hline \pounds 4.57 \end{array}$$

Alternatively, children can set the amounts to whole numbers, i.e.  $895 - 438$  and convert to pounds after the calculation.

***NB If children have reached the concise stage, they will then continue this method through into years 5 and 6. They will not go back to using the expanded methods.***

Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, counting on using an empty number line should be used.

$$511 - 197 = 314$$



## Year 5

### Rapid Recall of

#### Mental strategies

- use known number facts and place value to subtract pairs of three-digit multiples of 10 and two-digit numbers with one decimal place;
- subtract the nearest multiple of 10 or 100 then adjust;
- develop further the relationship between addition and subtraction.

#### Mental Calculations

- subtract any pair of three-digit multiples of 10, eg 620 - 380;
- subtract any pair of decimal fractions each with units and tenths, or each with tenths and hundredths, eg 0.63 - 0.48;
- subtract a four-digit number just less than a multiple of 1000 from a four-digit number just more than a multiple of 1000, eg 5001 - 1997;
- subtract numbers mentally with more than four digits.

## Y5 Written

### Decomposition

$$\begin{array}{r} 614 \text{ }^{14} \\ 7\cancel{1}4 \\ - 286 \\ \hline 468 \end{array}$$

Children should:

- ✓ be able to subtract numbers with different numbers of digits;
- ✓ begin to find the difference between two decimal fractions with up to three digits and the same number of decimal places;
- ✓ know that decimal points should line up under each other.

**NB** If your children have reached the concise stage they will then continue this method through into year 6. They will not go back to using the expanded methods.

## Year 6

### Rapid Recall of

#### Mental strategies

- consolidate all strategies from previous years;
- use knowledge of number facts and place value to subtract pairs of three-digit multiples of 10 and two-digit numbers with one decimal place;
- subtract the nearest multiple of 10, 100 or 1000, then adjust;
- continue to use the relationship between addition and subtraction.

#### Mental Calculations

### Y6 Written

#### Decomposition

$$\begin{array}{r} \phantom{0}^5 \phantom{0}^{13} \phantom{0}^{16} \\ 6467 \\ - 2684 \\ \hline 3783 \end{array}$$

Children should:

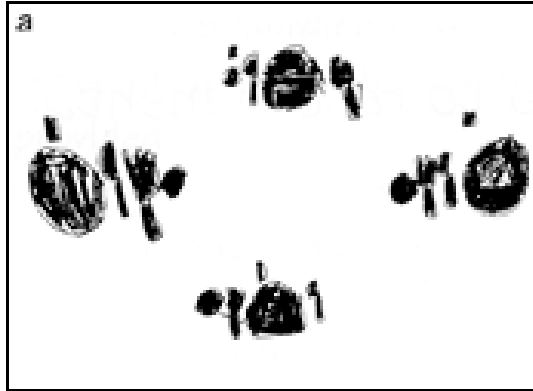
- ✓ be able to subtract numbers with different numbers of digits;
- ✓ be able to subtract two or more decimal fractions with up to three digits and either one or two decimal places;
- ✓ know that decimal points should line up under each other.

+ - + - + - + - + - + - +

## **Multiplication**

### **YR and Y1**

Children will experience equal groups of objects and will count in 2s, 10s and 5s. They will work on practical problem-solving activities involving equal sets or groups.



Children will work with concrete objects, make pictorial representations. With the support of the teacher the children will use arrays.

### **Y2**

#### **Rapid Recall of**

- multiplication facts for the 2, 5 and 10 times-tables;
- doubles of all numbers to ten;
- multiplication facts up to  $5 \times 5$ , eg  $4 \times 3$ .

#### **Mental strategies**

- use knowledge of number facts and place value to multiply by 2, 5 or 10; relate the 5 times table to the divisions on the clock face;
- use doubles and halves and halving as the inverse of doubling.

#### **Mental Calculations**

- halve any multiple of 10 up to 100, eg halve 50.

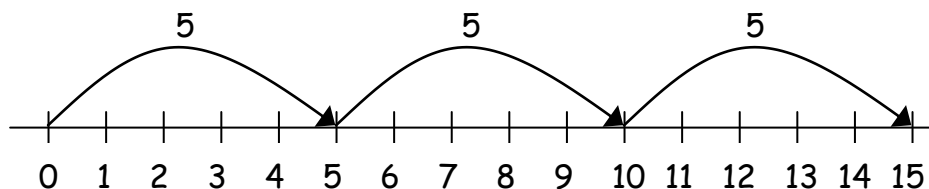
### **Step 2 (Y2) Written**

Children will develop their understanding of multiplication and use jottings to support calculation:

- ✓ **Repeated addition**

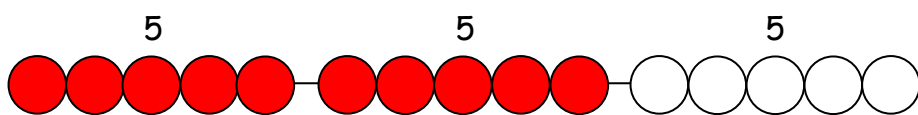
3 times 5 is  $5 + 5 + 5 = 15$  or 3 lots of 5 or  $5 \times 3$   
 Repeated addition can be shown easily on a number line:

$$5 \times 3 = 5 + 5 + 5$$



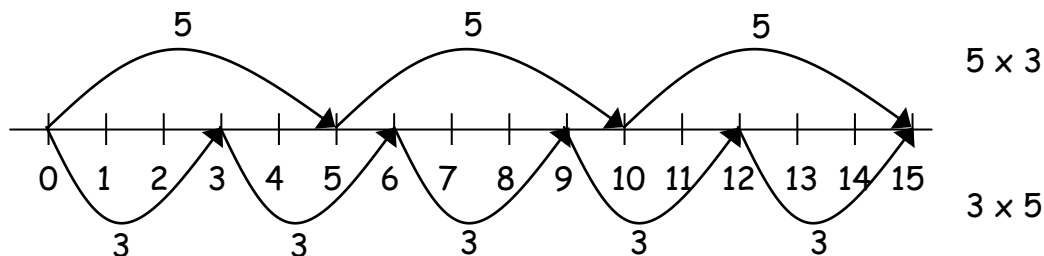
and on a bead bar:

$$5 \times 3 = 5 + 5 + 5$$



✓ **Commutativity**

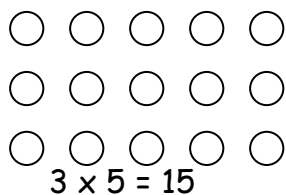
Children should know that  $3 \times 5$  has the same answer as  $5 \times 3$ . This can also be shown on the number line.



$$3 \times 5 = 3+3+3+3+3 = 15$$

✓ **Arrays**

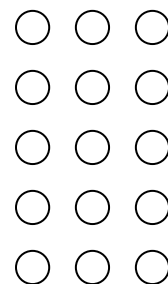
Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



$$5 \times 3 = 15$$

or

$$3 \times 5 = 15$$



$$3 \times 5 = 15$$

$$5 \times 3 = 15$$

## Y3

### Rapid Recall of

- multiplication facts for the 2, 3, 4, 5, (6), 8 and 10 times-tables.

### Mental strategies

- through doubling connect the 2, 4 and 8 times tables;
- to multiply a number by 10 or 100, shift its digits one or two places to the left;
- use knowledge of number facts and place value to multiply or divide by 2, 5, 10, 100;
- use doubling or halving;
- say or write a division statement corresponding to a given multiplication statement.

### Mental Calculations

- doubles:
- double any number to at least 20, eg double 18, and corresponding halves, eg halve 36;
- double 60;
- double 35;
- double 450;
- multiply single-digit numbers by 10 or 100, eg  $6 \times 100$ ;
- multiply a two-digit number by a single-digit number using mental methods, then progressing to formal written methods.

### Y3 Written

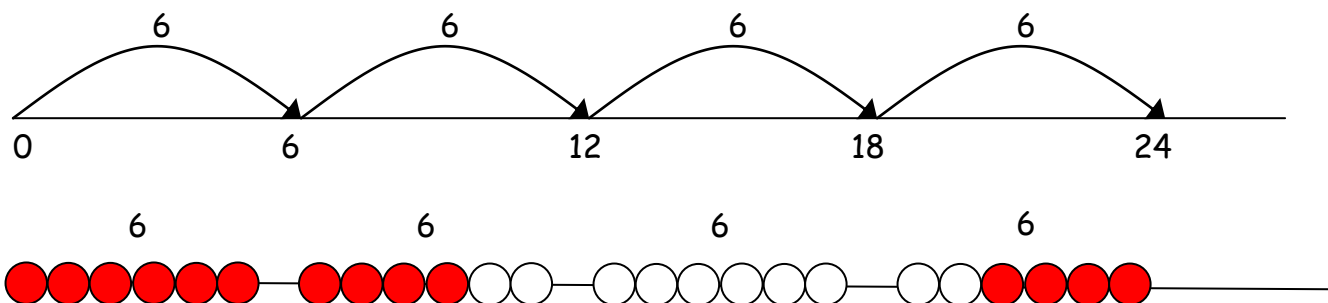
Children will continue to use:

✓ **Repeated addition**

4 times 6 is  $6 + 6 + 6 + 6 = 24$  or 4 lots of 6 or  $6 \times 4$

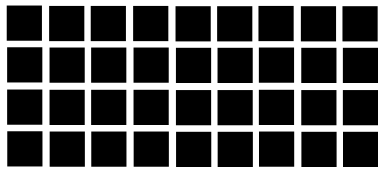
Children should use number lines or bead bars to support their understanding.

$6 \times 4$



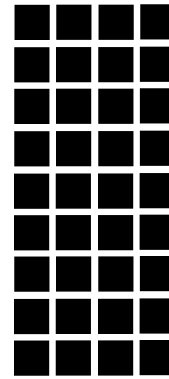
✓ **Arrays**

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



$4 \times 9 = 36$

$9 \times 4 = 36$



$9 \times 4 = 36$

$4 \times 9 = 36$

✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations**

$\square \times 5 = 20$

$3 \times \triangle = 18$

$\square \times \circ = 32$

✓ **Partitioning**

$$\begin{aligned} 38 \times 5 &= (30 \times 5) + (8 \times 5) \\ &= 150 + 40 \\ &= 190 \end{aligned}$$

✓ **Grid method**

$$\begin{array}{r} \times \quad 30 \quad 8 \\ 5 \quad \boxed{150} \quad \boxed{40} = 190 \end{array}$$

✓ **Formal written method**

If a child is confident with the grid method, they will move on to more formal written methods of multiplication

$$\begin{array}{r} \quad 3 \quad 8 \\ \times \quad \quad 5 \\ \hline \quad 4 \quad 0 \quad (8 \times 5) \\ 1 \quad 5 \quad 0 \quad (30 \times 5) \\ \hline 1 \quad 9 \quad 0 \end{array}$$

$$\begin{array}{r} \quad 3 \quad 8 \\ \times \quad \quad 5 \\ \hline 1 \quad 9 \quad 0 \\ \quad 4 \end{array}$$

## Y4

### Rapid Recall of

- multiplication facts to  $12 \times 12$ .

### Mental strategies

- double any two-digit number by doubling tens first;
- use known number facts and place value to multiply, including multiplying by 10 and then 100;
- partition to carry out multiplication;
- use doubling or halving;
- use closely-related facts to carry out multiplication and division;
- use the relationship between multiplication and division.

### Mental Calculations

- doubles and halves:
- double any whole number from 1 to 50, eg double 36;
- double any multiple of 10 to 500, eg  $380 \times 2$ ;
- double any multiple of 5 to 100, eg  $65 \times 2$ ;
- multiply any two-digit number by 10, eg  $26 \times 10$ ;
- multiply any two-digit multiple of 10 by 2, 3, 4 or 5, eg  $60 \times 4$ ,  $80 \times 3$ ;
- multiply three numbers together.

## Y4-Y6 Written

During years 4, 5 and 6, the main teaching of multiplication will be through use of the grid method. Once children are confident with the grid method, they will then quickly move on to more formal written methods. More able children will access the more formal methods at a quicker rate than their peers.

### **Grid method $TU \times U$**

(Short multiplication: multiplication by a single digit)

$$23 \times 8$$

Children will approximate first.

$23 \times 8$  is approximately  $25 \times 8 = 200$

$$\begin{array}{r} \times \quad 20 \quad 3 \\ 8 \quad \boxed{160} \quad \boxed{24} = 184 \end{array}$$

Remember to use the grid method with confidence



### Grid method HTU x U

(Short multiplication: multiplication by a single digit)

$$346 \times 9$$

Children will approximate first

$$346 \times 9 \text{ is approximately } 350 \times 10 = 3500$$

$$\begin{array}{r} \times \quad 300 \quad 40 \quad 6 \\ 9 \quad \boxed{2700} \quad \boxed{360} \quad \boxed{54} \end{array} = 3114$$

### TU x TU

(Long multiplication: multiplication by more than a single digit)

$$72 \times 38$$

Children will approximate first

$$72 \times 38 \text{ is approximately } 70 \times 40 = 2800$$

$$\begin{array}{r} \times \quad 70 \quad 2 \\ 30 \quad \boxed{2100} \quad \boxed{60} = 2160 \\ 8 \quad \boxed{560} \quad \boxed{16} = 576 \\ \hline 2736 \\ \phantom{2736} 1 \end{array}$$

*Using similar methods, they may be able to multiply decimals with one decimal place by a single-digit number, approximating first. They should know that the decimal points line up under each other.*

e.g.  $4.9 \times 3$

Children will approximate first

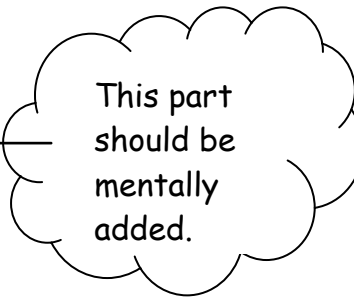
$$4.9 \times 3 \text{ is approximately } 5 \times 3 = 15$$

$$\begin{array}{r} \times \quad 4 \quad 0.9 \\ 3 \quad \boxed{12} \quad \boxed{2.7} \end{array} = 14.7$$

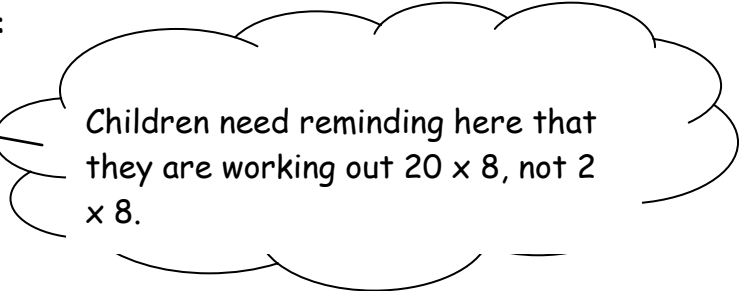
Once the children are confident with the grid method, they can move on to expanded and more compact methods for multiplication of TU x U, HTU x U, TU x TU.

In year 4 children will begin to show a formal method of recording money. More formal methods are introduced as the children are ready.

This would then move onto a long multiplication layout:

$$\begin{array}{r} 23 \\ \times 8 \\ \hline 24 \\ \hline 160 \\ \hline 184 \end{array}$$


This then moves onto a one-step approach:

$$\begin{array}{r} 23 \\ \times 8 \\ \hline 184 \\ 2 \end{array}$$


|   |                   |   |
|---|-------------------|---|
| $\begin{array}{r} 346 \\ \times 9 \\ \hline 54 \\ 360 \\ 2700 \\ \hline 3114 \\ 11 \end{array}$ | $\longrightarrow$ | $\begin{array}{r} 346 \\ \times 9 \\ \hline 3114 \\ 45 \end{array}$ |
|---|-------------------|---|

For these types of calculation (TU x TU or HTU x TU), children should stick to an expanded method until extremely confident.

|   |                   |   |   |   |   |   |  |  |   |   |   |  |   |   |  |   |   |   |   |   |   |   |   |   |   |   |  |   |  |  |
|---|-------------------|---|---|---|---|---|--|--|---|---|---|--|---|---|--|---|---|---|---|---|---|---|---|---|---|---|--|---|--|--|
| $\begin{array}{r} 72 \\ \times 38 \\ \hline 16 \\ 560 \\ 60 \\ \hline 2100 \\ \hline 2736 \\ 1 \end{array}$ | $\longrightarrow$ | <table style="border-collapse: collapse; margin: auto;"> <tr> <td></td> <td style="background-color: yellow;">5</td> <td style="background-color: yellow;">1</td> <td style="background-color: yellow;">?</td> </tr> <tr> <td></td> <td></td> <td>7</td> <td>2</td> </tr> <tr> <td style="text-align: right;">x</td> <td></td> <td>3</td> <td>8</td> </tr> <tr> <td></td> <td style="border-top: 1px solid black;">5</td> <td style="border-top: 1px solid black;">7</td> <td style="border-top: 1px solid black;">6</td> </tr> <tr> <td style="text-align: right;">2</td> <td>1</td> <td>6</td> <td>0</td> </tr> <tr> <td style="text-align: right;">2</td> <td style="border-top: 1px solid black;">7</td> <td style="border-top: 1px solid black;">3</td> <td style="border-top: 1px solid black;">6</td> </tr> <tr> <td></td> <td>1</td> <td></td> <td></td> </tr> </table> |   | 5 | 1 | ? |  |  | 7 | 2 | x |  | 3 | 8 |  | 5 | 7 | 6 | 2 | 1 | 6 | 0 | 2 | 7 | 3 | 6 |  | 1 |  |  |
|   | 5                 | 1   | ? |   |   |   |  |  |   |   |   |  |   |   |  |   |   |   |   |   |   |   |   |   |   |   |  |   |  |  |
|   |                   | 7   | 2 |   |   |   |  |  |   |   |   |  |   |   |  |   |   |   |   |   |   |   |   |   |   |   |  |   |  |  |
| x   |                   | 3   | 8 |   |   |   |  |  |   |   |   |  |   |   |  |   |   |   |   |   |   |   |   |   |   |   |  |   |  |  |
|   | 5                 | 7   | 6 |   |   |   |  |  |   |   |   |  |   |   |  |   |   |   |   |   |   |   |   |   |   |   |  |   |  |  |
| 2   | 1                 | 6   | 0 |   |   |   |  |  |   |   |   |  |   |   |  |   |   |   |   |   |   |   |   |   |   |   |  |   |  |  |
| 2   | 7                 | 3   | 6 |   |   |   |  |  |   |   |   |  |   |   |  |   |   |   |   |   |   |   |   |   |   |   |  |   |  |  |
|   | 1                 |   |   |   |   |   |  |  |   |   |   |  |   |   |  |   |   |   |   |   |   |   |   |   |   |   |  |   |  |  |

## Y5

### Rapid Recall of

- multiplication facts to  $12 \times 12$ ;
- recall prime numbers up to 19;
- identify squared and cubed numbers.

### Mental strategies

- use factors and find all factor pairs of a number;
- find common factors of two numbers;
- partition to carry out multiplication;
- use doubling;
- use closely-related facts to carry out multiplication and division;
- use the relationship between multiplication and division;
- use knowledge of number facts and place value to multiply.

### Mental Calculations

- multiply any two- or three-digit number by 10 or 100, eg  $79 \times 100$ ,  $363 \times 100$ ;
- multiply any two-digit multiple of 10 by a single-digit, eg  $60 \times 7$ ,  $90 \times 6$ ;
- double any whole number from 1 to 100, multiples of 10 to 1000;

## Y6

### Rapid Recall of

- squares of all integers from 1 to 12.

### Mental strategies

- use factors;
- partition to carry out multiplication;
- use doubling;
- use closely-related facts to carry out multiplication and division;
- use the relationship between multiplication and division;
- use knowledge of number facts and place value to multiply or divide.

### Mental Calculations

- multiply any two-digit number by a single-digit, eg  $34 \times 6$ ;
- multiply any two-digit number by 50 or 25, eg  $23 \times 50$ ,  $47 \times 25$ ;
- multiply any whole number by 10 or 100;
- find squares of multiples of 10 to 100;
- find any multiple of 10% of a whole number or quantity, eg 70% of £20, 50% of 5kg, 20% of 2 metres.

## Y6 Written

In Year 6 no new methods will be taught, although extensions for difficulty should be taught, e.g. ThHTU × U, HTU × TU, U.th × U and ThHTU × TU.

These will be taught using the same methods as those from Year 4.

### Further example

24 × 16 becomes

$$\begin{array}{r} \phantom{2} 2 \phantom{4} \\ \phantom{2} \mathbf{2} \phantom{4} \mathbf{4} \\ \times \phantom{2} \mathbf{1} \phantom{4} \mathbf{6} \\ \hline \mathbf{2} \phantom{4} \mathbf{4} \mathbf{0} \\ \mathbf{1} \phantom{4} \mathbf{4} \phantom{4} \\ \hline \mathbf{3} \phantom{4} \mathbf{8} \phantom{4} \\ \hline \end{array}$$

Answer: 384

124 × 26 becomes

$$\begin{array}{r} \phantom{1} \phantom{2} \\ \phantom{1} \phantom{2} \mathbf{4} \\ \times \phantom{1} \phantom{2} \mathbf{2} \phantom{6} \\ \hline \mathbf{2} \phantom{4} \mathbf{8} \phantom{0} \\ \phantom{2} \mathbf{7} \phantom{4} \phantom{4} \\ \hline \mathbf{3} \phantom{2} \mathbf{2} \phantom{2} \mathbf{4} \\ \hline \phantom{1} \phantom{1} \\ \hline \end{array}$$

Answer: 3224

124 × 26 becomes

$$\begin{array}{r} \phantom{1} \phantom{2} \\ \phantom{1} \phantom{2} \mathbf{4} \\ \times \phantom{1} \phantom{2} \mathbf{2} \phantom{6} \\ \hline \phantom{2} \mathbf{7} \phantom{4} \phantom{4} \\ \mathbf{2} \phantom{4} \mathbf{8} \phantom{0} \\ \hline \mathbf{3} \phantom{2} \mathbf{2} \phantom{2} \mathbf{4} \\ \hline \phantom{1} \phantom{1} \\ \hline \end{array}$$

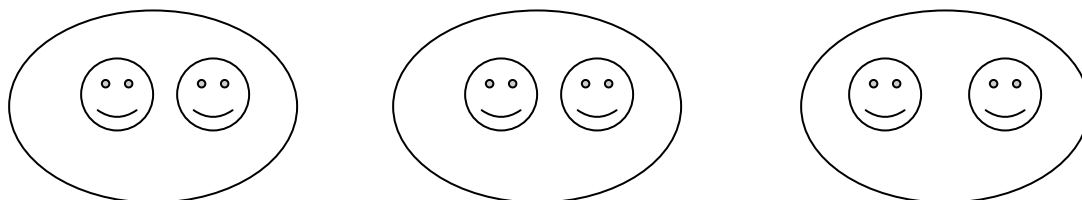
Answer: 3224

+ - + - + - + - + - + - +

## Division

### YR and Y1

Children will understand equal groups and share items out in play and problem-solving. They will count in 2s and 10s and later in 5s.



Children will work with concrete objects, make pictorial representations. With the support of the teacher the children will use arrays. Children will find half of a quantity.

### Y2

#### Rapid Recall of

- multiplication facts for the 2, 5 and 10 times-tables and **corresponding division facts**;  
relate the 5 times table to the divisions on the clock face;
- doubles of all numbers to ten and the **corresponding halves**.

#### Mental strategies

- use knowledge of number facts and place value to divide by 2, 5 or 10;
- use doubles and halves and halving as the inverse of doubling.

#### Mental Calculations

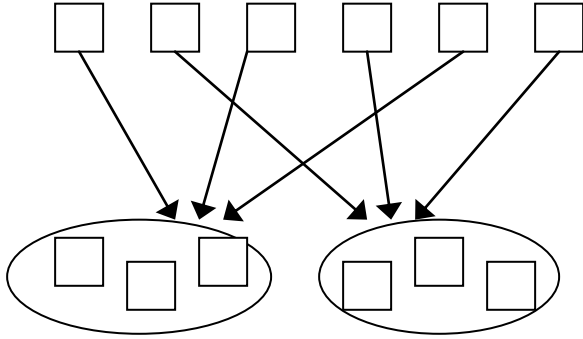
- halve any multiple of 10 up to 100, eg halve 50.

### Y2 Written

Children will develop their understanding of division and use jottings to support calculation

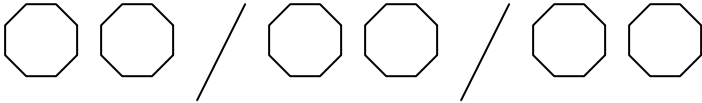
- ✓ **Sharing equally**

6 sweets shared between 2 people; how many do they each get?



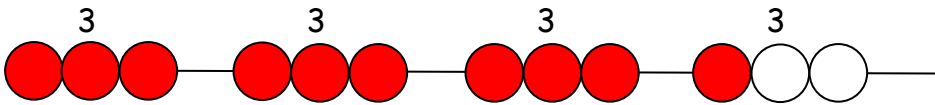
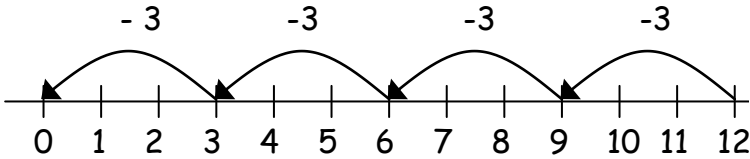
✓ **Grouping or repeated subtraction**

There are 6 sweets, how many people can have 2 sweets each?



✓ **Repeated subtraction using a number line or bead bar (ITP)**

$$12 \div 3 = 4$$



The bead bar may help children with interpreting division calculations.

✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations**

$$\square \div 2 = 4 \qquad 20 \div \triangle = 4 \qquad \square \div \triangle = 4$$

Children need to be shown that division can't be done in any order, unlike multiplication.

## Y3

### Rapid Recall of

- multiplication facts for the 2,3,4, 5, 6, 8 and 10 times-tables and **corresponding division facts**.

### Mental strategies

- use knowledge of number facts and place value to divide by 2, 5, 10, 100;
- use halving;
- say or write a division statement corresponding to a given multiplication statement.

### Mental Calculations

- doubles:
- double any number to at least 20, eg double 18, and corresponding halves, eg halve 36;
- double 60, halve 120;
- double 35, halve 70;
- = double 450, halve 900;
- divide any multiple of 10 by 10, eg  $60 \div 10$ , and any multiple of 100 by 100, eg  $700 \div 100$ .

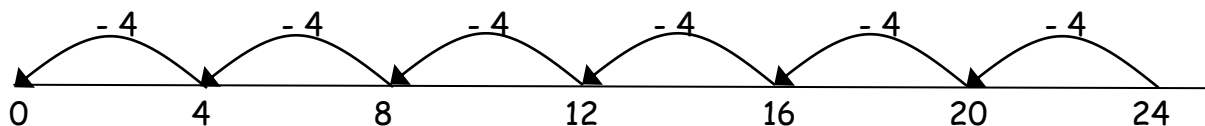
## Y3 Written

Ensure that the emphasis in Y3 is on grouping rather than sharing.  
Children will continue to use:

- ✓ **Repeated subtraction using a number line**

Children will use an empty number line to support their calculation.

$$24 \div 4 = 6$$

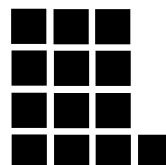
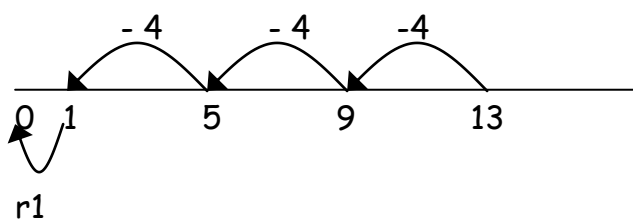


Children should also move onto calculations involving remainders.

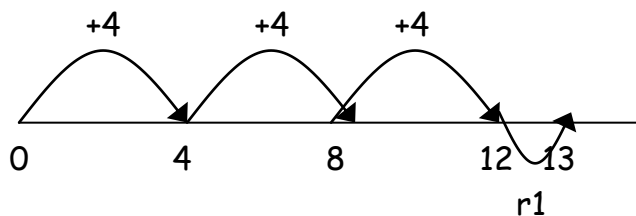
$$13 \div 4 = 3 \text{ r } 1$$

or

$$13 \div 4 = 3 \text{ r } 1$$



If children are confident with their tables, they may prefer a method of counting on.



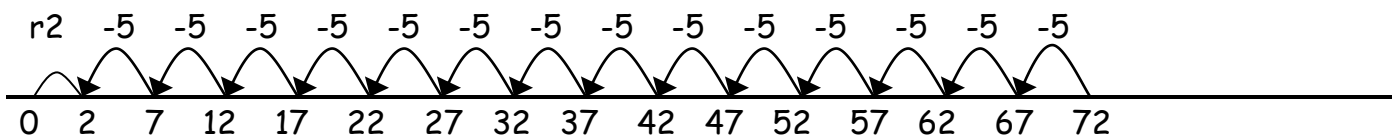
✓ Using symbols to stand for unknown numbers to complete equations using inverse operations

$26 \div 2 = \square$        $24 \div \triangle = 12$        $\square \div 10 = 8$

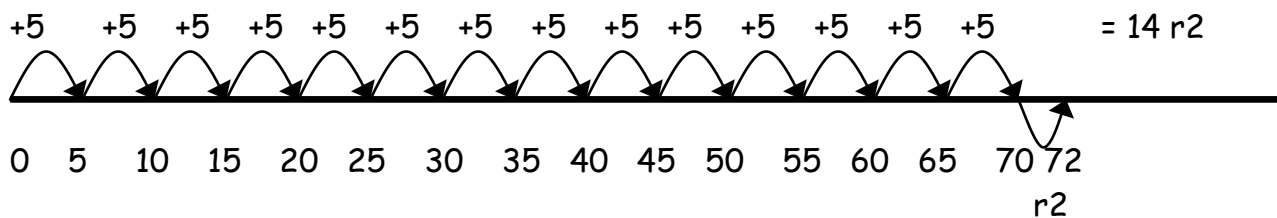
Children will move from mental calculation of two digits divided by a single digit to a more formal method.

Children will develop their use of repeated subtraction to be able to subtract multiples of the divisor. Initially, these should be multiples of 10s, 5s, 2s and 1s—numbers with which the children are more familiar.

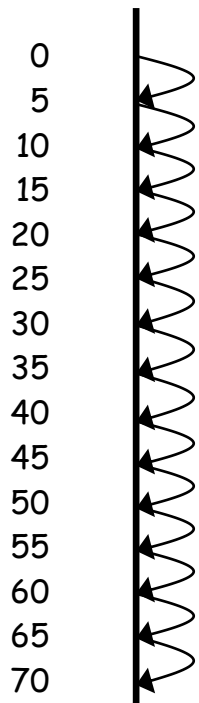
$72 \div 5 = 14 \text{ r}2$



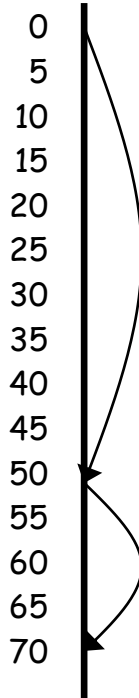
If children are confident with their tables, they may prefer a method of counting on.







From small steps to chunking



$$\underline{10} \times 5$$

$$\underline{4} \times 5$$

$$\underline{10} + \underline{4} = 14$$

So  $70 \div 5 = 14$   
because  
 $14 \times 5 = 70$

Then onto the vertical method:

**Chunking**

$$\begin{array}{r} 24 \\ 3 \overline{) 72} \\ \underline{- 30} \quad (3 \times 10) \\ 42 \\ \underline{- 30} \quad (3 \times 10) \\ 12 \quad (3 \times 4) \end{array}$$

**Top tip:** Children to subtract chunks they are most comfortable with  
Work out 2x  
5x  
10x

Moving on to  
**Short division TU ÷ U**

$72 \div 3$

$$\begin{array}{r} 24 \\ 3 \overline{) 72} \end{array}$$

Make sure children understand language of what they are dividing, eg how many 3s are in 70. A link to multiplication would be beneficial here.

## Y4

### Rapid Recall of

- division facts corresponding to tables up to  $12 \times 12$ .

### Mental strategies

- use known number facts and place value to divide, including dividing by 10 and then 100;
- use halving;
- use closely-related facts to carry out division;
- use the relationship between multiplication and division;
- understand what happens when you divide by 1.

### Mental Calculations

- doubles and halves;
- double any whole number from 1 to 50, eg double 36, and find all the corresponding halves, eg  $96 \div 2$ ;
- double any multiple of 10 to 500, eg  $380 \times 2$ , and find all the corresponding halves, eg  $760 \div 2$ ,  $130 \div 2$ ;
- divide a multiple of 100 by 10, eg  $600 \div 10$ .

## Y4 Written

To try and become fluent in the formal method of short division with exact answers.

## Y5

### Rapid Recall of

- division facts corresponding to tables up to  $12 \times 12$ ;
- factors of all numbers.

### Mental strategies

- use halving;
- use closely-related facts to carry out multiplication and division;
- use the relationship between multiplication and division;
- use knowledge of number facts and place value to divide.

### Mental Calculations

- divide a multiple of 100 by 10 or 100, eg  $4000 \div 10$ ,  $3600 \div 100$ ;
- double any whole number from 1 to 100, multiples of 10 to 1000, and find corresponding halves;
- find 50%, 25%, 10% of a small whole numbers or quantities, eg 25% of £8;
- divide whole numbers and those involving decimals by 10, 100 and 1000.

## Y5 Written

Using **chunking** children can start to subtract larger multiples of the divisor, e.g. 30x and then move on to use written methods to solve short division  $\text{ThHTU} \div \text{U}$ .

### Short division $\text{HTU} \div \text{U}$

$$196 \div 6$$

$$\begin{array}{r} 32 \text{ r } 4 \\ 6 \overline{) 196} \end{array}$$

Again, the language is very important here, with the link to multiplication.

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

Children need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division. For example  $240 \div 52$  is 4 remainder 32, but whether the answer should be rounded up to 5 or rounded down to 4 depends on the context.

### Short division $\text{ThHTU} \div \text{U}$

$$2453 \div 5$$

$$\begin{array}{r} \underline{0480} \text{ r } 3 \\ 5 \overline{) 22453} \end{array} \quad \text{becomes} \quad \begin{array}{r} \underline{04806} \\ 5 \overline{) 22453.0} \end{array}$$

Pupils need to express the results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding.

$$98 \div 4 = 98/4 = 24 \text{ r } 2 = 24\frac{1}{2} = 24.5 = 25$$

## Y6

### Rapid Recall of

#### Mental strategies

- use halving;
- use closely-related facts to carry out multiplication and division;
- use the relationship between multiplication and division;
- use knowledge of number facts and place value to multiply or divide.

## Mental Calculations

- divide any whole number by 10 or 100, giving any remainder as a decimal, eg  $47 \div 10 = 4.7$ ,  $1763 \div 100 = 17.63$ ;
- find any multiple of 10% of a whole number or quantity, eg 70% of £20, 50% of 5kg, 20% of 2 metres.

## Y6 Written

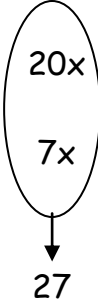
Children will continue to use written methods to solve short division  $TU \div U$  and  $HTU \div U$ .

### Long division $HTU \div TU$

$$972 \div 36$$

$$\begin{array}{r} 27 \\ 36 \overline{) 972} \\ \underline{- 720} \\ 252 \\ \underline{- 252} \\ 0 \end{array}$$

Answer : 27



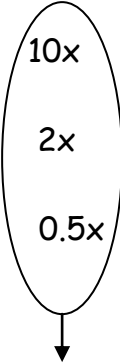
Any **remainders** should be shown as **fractions**, i.e. if the children were dividing 32 by 10, the answer should be shown as  $3 \frac{2}{10}$  which could then be written as  $3 \frac{1}{5}$  in its lowest terms.

Extend to decimals with up to two decimal places. Children should know that decimal points line up under each other.

$$87.5 \div 7$$

$$\begin{array}{r} 12.5 \\ 7 \overline{) 87.5} \\ \underline{- 70.0} \\ 17.5 \\ \underline{- 14.0} \\ 3.5 \\ \underline{- 3.5} \\ 0 \end{array}$$

Answer : 12.5



Further example from Appendix 1

432 ÷ 15 becomes

$$\begin{array}{r}
 28 \text{ r } 12 \\
 15 \overline{)432} \\
 \underline{30 \phantom{0}} \\
 132 \\
 \underline{120 \phantom{0}} \\
 120 \\
 \underline{120} \\
 0
 \end{array}$$

Answer: 28 remainder 12

432 ÷ 15 becomes

$$\begin{array}{r}
 28 \\
 15 \overline{)432} \\
 \underline{30 \phantom{0}} \quad 15 \times 20 \\
 132 \\
 \underline{120 \phantom{0}} \quad 15 \times 8 \\
 120 \\
 \underline{120} \\
 0
 \end{array}$$

$$\frac{\cancel{12}}{\cancel{15}} = \frac{4}{5}$$

Answer:  $28 \frac{4}{5}$

432 ÷ 15 becomes

$$\begin{array}{r}
 28.8 \\
 15 \overline{)432.0} \\
 \underline{30 \phantom{0} \phantom{0}} \\
 132 \\
 \underline{120 \phantom{0}} \\
 120 \\
 \underline{120} \\
 0 \\
 \phantom{0} \phantom{0} \\
 \underline{\phantom{0} \phantom{0}} \\
 0
 \end{array}$$

Answer: 28.8

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By the end of year 6 children will have a range of calculation methods, both mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

- 1) they are not ready;
- 2) they are not confident.

Children should be encouraged to approximate their answers before calculating.

Children should be encouraged to check their answers after calculation using an appropriate strategy.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.